# AXISYMMETRIC DETACHED FLOW PAST A SOLID OF REVOLUTION WITH SMALL CAVITATION NUMBERS

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In [1] Grigorian considered the problem of detached flow past a slender solid of revolution by an ideal incompressible fluid at zero angle of attack and with small cavitation numbers. We shall use more exact estimates to derive a differential equation for the source density on the axis of symmetry. Though similar to the expression obtained in [1], our equation contains additional terms. The problem reduces to our differential equation only if we make additional assumptions (i.e. over and above those of [1]) about the order of smallness of the cavitation number $\sigma$. If such assumptions are not made, it reduces to an integro-differential equation.

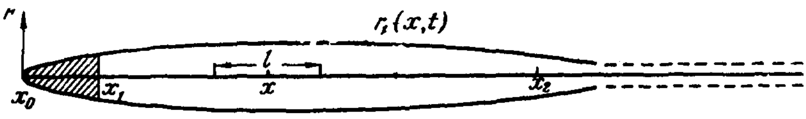


Fig. 1

Let us consider the flow diagram shown in Fig. 1. Here $x_0$, $x_1$ is the body; $x_1$, $x_2$ is the cavity; $r_1$ $(x, t)$ is the boundary of the cavity; $x_2$, $+\infty$ is the wake behind the body and cavity; $V(t)$ is the absolute velocity of the body; $P$ is the pressure inside the cavity; $P_\infty$ is the pressure at infinity; the velocity at infinity is equal to zero.

We assume that $r$, $x$, $V$, $P$, and $t$ are already dimensionless, so that their unity values are associated with some $L_0$, $V_0$, $P_0 = \rho_0 V_0^2$, $t_0 = L_0 / |V_0|$, where $V_0$ is the velocity of the body at a given instant; $\rho_0$ is the density of the fluid.

We shall attempt to find the dimensionless potential of the velocity field beyond the boundaries of the body, cavity, and wake in the form

$$\varphi(r, x, t) = \int_{x_1}^{\infty} \frac{g(\xi, t)\, d\xi}{\sqrt{(\xi - x)^2 + r^2}} \tag{1}$$

We assume that the cavity is slender at the point $x \sim 1$, i.e. that at this point

$$r_1 \lesssim \varepsilon \tag{2}$$

In addition, we assume tentatively the validity of the estimates

$$|g| \lesssim \varepsilon^2 \tag{3}$$

$$|g_t|, \ |g_{tx}| \lesssim \varepsilon^2 \qquad \text{for } x_1 < x < x_2 \tag{4}$$

and that $g$ tends to zero quite rapidly as $x \to \infty$.

From (1) we have

$$\varphi_t(r_k, x, t) = -\frac{g(x_0 t)\, x_0^{\cdot}}{\sqrt{(x_0-x)^2+r_1^2}} + \int_{x_0}^{x_1} \frac{g_t d\xi}{\sqrt{(\xi-x)^2+r_1^2}} + \int_{x_1}^{x-l/2} \frac{g_t d\xi}{\sqrt{(\xi-x)^2+r_1^2}} +$$

$$+ \int_{x-l/r}^{x+l/r} \frac{g_t d\xi}{\sqrt{(\xi-x)^2+r_1^2}} + \int_{x+l/2}^{x_2} \frac{g_t d\xi}{\sqrt{(\xi-x)^2+r_1^2}} + \int_{x_2}^{\infty} \frac{g_t d\xi}{\sqrt{(\xi-x)^2+r_1^2}} = \tag{5}$$

$$= -\frac{g(x_0, t)}{\sqrt{(x_0-x)^2+r_1^2}} + J_1 + J_2 + J_3 + J_4 + J_5 \left( l = \frac{1}{\ln^2 \varepsilon}\,,\ x_0^{\cdot} = -1 \right)$$

To within higher-order small quantities we have

$$\left| \frac{g(x_0, t)}{\sqrt{(x_0-x)^2+r_1^2}} \right| = \frac{|g(x_0, t)|}{|x_0 - x|} \approx \varepsilon^2, \qquad |J_1| = \left| \int_{x_1}^{x} \frac{g_t\, d\xi}{|x-\xi|} \right| \lesssim \varepsilon^2 \tag{6}$$

$$|J_2| = \left| \int_{x_1}^{x-\frac{1}{2}l} \frac{g_t d\xi}{|x-\xi|} \right| \leqslant |\max g_t| \ln \left| \frac{2(x_1-x)}{l} \right| \lesssim \varepsilon^2\, 2\, |\ln |\ln \varepsilon|| \tag{7}$$

Similarly, we find that the integrals $J_4$ and $J_3$ conform to the relations

$$|J_4| \lesssim \varepsilon^2\, 2 \ln |\ln \varepsilon|$$

$$J_3 = [g_t(x, t) + \delta] \int_{x-\frac{1}{2}l}^{x+\frac{1}{2}l} \frac{d\xi}{\sqrt{(\xi-x)^2+r_1^2}} = (g_t + \delta) \ln \left[ \frac{1 + \sqrt{1+4(r/l)^2}}{-1+\sqrt{1+4(r/l)^2}} \right] =$$

$$= -2g_t \ln r + 2g_t \ln l - 2\delta \ln(r/l) = -2g_t \ln r \tag{8}$$

$$|\delta| \leqslant |\max g_{tx}|\, l \leqslant \frac{\varepsilon^2}{|\ln \varepsilon|}$$

By hypothesis, the integral $J_5$ satisfies the relation

$$J_5 \sim |\max g_t| \lesssim \varepsilon^2 \tag{9}$$

Similarly, for $V_r^2$ and $V^2$ we have the estimates

$$\varphi_r = -\frac{2g(x, t)}{r}, \qquad (\varphi_x)^2 \sim g^2 \lesssim \varepsilon^4 \tag{10}$$

The boundary conditions at the cavity boundary are the kinematic condition and the condition of constant pressure,

$$\partial r_1 / dt = \varphi_r, \qquad P_\infty - P_1 = \varphi_t + \tfrac{1}{2}[\varphi_r^2 + \varphi_x^2] \tag{11}$$

From the first equation of (11) we have

$$g = -\frac{1}{4}\frac{\partial r_1^2}{\partial t} = -\frac{1}{4} u_t \qquad u = r_1^2 \tag{12}$$

Substituting (6), (7), (8), and (10) into the second relation of (11) and recalling (12), we obtain

$$\frac{1}{4} u_{tt} \ln u + \frac{1}{8}\frac{(u_t)^2}{u} + \frac{g(x_0, t)}{|x-x_0|} + J_1 + J_5 + J_2 + J_4 - \frac{1}{2}\sigma = 0 \tag{13}$$

$$\sigma = \frac{P_\infty - P_1}{\frac{1}{2}\rho_0 V_0^2} = 2(p_\infty - p_1) \tag{14}$$

Here $\sigma$ is the cavitation number. All the terms of integro-differential equation (13) with the exception of the first cavitation number and $(J_2 + J_4)$ have been estimated by means of the quantity $\epsilon^2$. Assuming that $\sigma \lesssim \varepsilon^2$, and recalling the estimate for $(J_2 + J_4)$, we find that

$$|u_{tt} \ln u| \lesssim \varepsilon^2 \ln|\ln \varepsilon|, \qquad \text{or} \qquad |u_{tt}| \lesssim \frac{\varepsilon^2 \ln|\ln \varepsilon|}{|\ln \varepsilon|} \tag{15}$$

Next, substituting the estimate $g_t \approx u_{tt}$ into (7), we obtain in its place the expression

$$|J_2 + J_4| \leqslant \frac{\varepsilon^2 (\ln|\ln \varepsilon|)^2}{|\ln \varepsilon|} \leqslant \varepsilon^2 \tag{16}$$

Hence, instead of (15) we can take the stronger inequality

$$|u_{tt} \ln \varepsilon| \lesssim \varepsilon^2, \qquad \text{or} \qquad |u_{tt}| \lesssim \frac{\varepsilon^2}{|\ln \varepsilon|} \tag{17}$$

This means that the estimate for (7) can be strengthened still further, and that $(j_2 + J_4)$ can be neglected in (13).

As a result we obtain the following differential equation at the cavity boundary:

$$u_{tt} \ln + \frac{1}{2} \frac{(u_t)^2}{u} - 2\sigma = -\frac{4g(x_0, t)}{|x - x_0|} - 4 \int\limits_{x_0}^{x_1} \frac{g_t d\xi}{|x - \xi|} - 4 \int\limits_{x_2}^{\infty} \frac{g_t d\xi}{|x - \xi|} \tag{18}$$

The additional conditions for this equation are the conditions for $x = x_1$ and the initial data. The $g(x, t)$ for $x_0 \leqslant x \leqslant x_1$ and $x_2 < x\ \infty$ must be prescribed on the basis of additional considerations (e.g. the equations of motion of slender bodies).

For $\sigma = 0$ and for steady motion $u(x, t) = u(x + t)$. Eq. (18) yields the following familiar asymptotic expression for the cavity boundary [2]:

$$u = \frac{c}{\sqrt{\ln x}}\, x$$

In the case of steady motion and a Riabushinskii flow diagram with conical cavitator and contractor ($g_x = \text{const}$, $g(0) = 0$) Eq. (18) becomes

$$u'' \ln u + \frac{1}{2} \frac{(u')^2}{u} - 2\sigma = \frac{2u(x_1)}{(x_1 - x_0)} \ln \frac{x - x_0 + \sqrt{(x - x_0)^2 + u}}{x - x_1 + \sqrt{(x - x_1)^2 + u}} +$$

$$+ \frac{2u(x_2)}{(x_3 - x_2)^2} \ln \frac{x_3 - x - \sqrt{(x_3 - x)^2 + u}}{x_2 - x + \sqrt{(x_2 - x)^2 + n}} \tag{19}$$

with boundary conditions expressing the continuity of the derivative at the streamline at the head and tail of the cavity.

$$u'(x_1) - \frac{2u(x_1)}{x_1 - x_0} = 0, \qquad u'(x_2) + \frac{2u(x_2)}{x_3 - x_2} = 0 \tag{20}$$

Numerical computations using (19) and (20) indicate that for an asymmetric Riabushinskii diagram with cavitator and contractor lengths differing by a factor of two and for $\sigma < 0.01$, the sum of forces acting on the cavitator and contractor is smaller than one-tenth of the cavitator drag. Exact solution, on the other hand, indicates that the d'Alembert paradox must hold.

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# ON A PLASTIC SHEAR WAVE

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A self-similar solution of the problem of propagation of a perturbation produced by a glancing collision against the boundary of a half-space whose material conforms to the Prandtl-Reuss equations is constructed.

Simple conditions of solvability of the problem for two types of boundary conditions are constructed. These boundary conditions correspond to the cases of 1) total adhesion and 2) Coulomb dry friction.

1. The Prandtl-Reuss equations are sometimes used to describe the motion of a soil under large loads [1]. Problems of this type usually contain two space variables and time, and can only be solved numerically. In some such problems it is necessary to consider the interaction of waves with a hard surface. The boundary conditions which this requires have not been investigated sufficiently.

It is natural to attempt to gain insight into the situation by way of some simple problem. We shall consider an elementary case which nevertheless retains some of the salient features of complex problems of wave and surface interaction.

Let a hard slab be pressed by the force $\sigma_0$ against the boundary of a half-space. At $t = 0$ the slab is set in motion with the constant velocity $v_0$ directed along the boundary.

For $t < 0$ the half-space is a rest, and the stress it experiences is constant.

Since the basic equations allow for the appearance of tangent stresses in the medium, we can stipulate at the boundary either an adhesion condition or the dry friction law natural in solid body contact.

In Section 2 we shall show that under the adhesion condition the problem has a solution only for velocities restricted by the inequality $v_0 \leq v_*$ ; a unique solution does not exist for $v_0 > v_*$. It will be shown that a solution exists only if the coefficient satisfies some (quite simple) inequality.

The notation is as follows: $x$ is a coordinate (the $x$-axis is directed into the half-space); $u$ is the velocity along $x$; $v$ is the velocity along the normal to $x$; $K$ is the bulk modulus; $G$ is the shear modulus; $\theta$ is the volume compression; $\sigma$ is the stress along $x$; $\tau$ is the tangent stress; $p$ is the hydrostatic pressure; $f$ is the coefficient of friction. The plasticity condition is